Transforming squeezed light into large amplitude Schrödinger cat states

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A quantum superposition of two coherent states of light, a so-called Schrödinger cat, or kitten, state, with small amplitude can be obtained by subtracting a photon from a squeezed vacuum state. In experiments this preparation can be made conditioned on the detection of a photon in the field from a squeezed light source. We propose and analyze an extended measurement strategy which allows generation of high fidelity cat states with larger coherent amplitude.

I. INTRODUCTION

A Schrödinger cat state is a quantum superposition of macroscopically distinguishable states. In quantum optics there is tradition to use the term for superpositions of different *coherent states* $|\alpha_1\rangle$ and $|\alpha_2\rangle$ such as the *even* $(+)$ and *odd* $(-)$ cat states

$$|\psi_{\text{cat}}^{\pm}\rangle = \frac{1}{\sqrt{2(1 \pm e^{-2|\alpha|^2})}} (|\alpha\rangle \pm |-\alpha\rangle). \qquad (1)$$

The odd cat state populates only odd photon number states, while the even cat state populates only even photon number states. A squeezed vacuum state, generated, e.g., in a degenerate optical parametric oscillator (OPO), is a superposition of even photon number states, and it has been suggested [1] to generate approximate odd cat states by subtracting a photon from such a state, i.e., by applying the photon annihilation operator $\hat{a}$ to a squeezed vacuum. As discussed in Refs. [1, 2, 3, 4] the resulting states are close to odd cat states when the cat size $|\alpha|$ is small, because both the odd cat and the photon subtracted squeezed vacuum state approach a single-photon state in the limit of very small $|\alpha|$ and very little squeezing, and for small amplitudes the degree of squeezing can be adjusted such that the ratio between the $n = 1$ and $n = 3$ number state components of the photon subtracted field match the ratio of the odd cat state.

Schrödinger cat states are interesting probes for quantum mechanical behavior at the mesoscopic and macroscopic level, and they may be used to investigate the role of decoherence in large systems. They have potential applications for high precision probing, and in particular they are useful resource states in optical quantum computing proposals which make use of linear optics and photon counting [5]. The application in quantum computing is particularly interesting, because the encoding of qubits in coherent states (and the resulting need for their superposition states, i.e., the cat states) only requires relatively small coherent state amplitudes to be sufficiently distinguishable by homodyne detection. For more detailed discussions of single- and two-qubit measurements in the coherent state and in the odd-even cat state basis, see [5], where it also follows that resource cat states with amplitudes of about $|\alpha| = \sqrt{6} \approx 2.45$ or larger are sufficient for quantum computing. Small cats have been generated experimentally using the method described above [6, 7, 8, 9], but the theoretically obtainable fidelity drops below 90%, when $|\alpha|$ exceeds 1.9, and other means have to be applied to make larger cats with high fidelity. It has thus been proposed to generate cats of larger size by combining smaller cats on beam splitters [10] or by amplifying cat states in an optical parametric amplifier [11].

We suggest here an approach, which heralds the production of larger cat states by a number of photo detection events. Dakna *et al.* [1] considered the states conditioned on multiple photo detection events, and theory [12] and experiments [13] have demonstrated the production of two-photon states in a signal beam conditioned on the detection of two idler photons from a nondegenerate OPO. Here we shall combine the field from the degenerate OPO with a coherent state field prior to counting of the photon numbers, as this allows us to effectively produce states which are mathematically equivalent to the result of applying operators of the form $\hat{O}_A = \prod_{i=1}^{A}(\hat{a} - \beta_i)$ to a squeezed vacuum state, where β_i, $i = 1, \ldots, A$, are adjustable complex numbers. Restricting the values of β_i so that one vanishes and the others occur in pairs $\pm\beta$ (for A odd), we can rewrite the operator $\hat{O}_A$ as $\hat{O}_{2k+1} = \left(\prod_{i=1}^{k}\left(\hat{a}^2 - \beta_i^2\right)\right)\hat{a}$, and the resulting state is a superposition of odd photon number states, but we now have k free complex parameters in addition to the squeezing parameter, which may be chosen to match more closely the number state amplitudes to the ones of the odd cat state. Similarly, an approximate even cat state may be produced by applying the operator $\hat{O}_{2k} = \prod_{i=1}^{k}\left(\hat{a}^2 - \beta_i^2\right)$ to a squeezed vacuum state. Application of $\hat{O}_{2k+1}$ ($\hat{O}_{2k}$) involves annihilation of $2k+1$ ($2k$) photons and since the probability to obtain each annihilation in a real experiment is small, we shall mainly be concerned with the case $k = 1$ below. In Sec. II we compute the cat state fidelities obtained for $k = 1$ and compare the results with the fidelities obtained for $k = 0$. In Sec. III we discuss how the operator $\hat{O}_{2k+1}$ may be approximately realized experimentally, and Sec. IV concludes the paper.

II. CAT STATE FIDELITY

We first determine the odd cat state fidelity for the case of a single photo detection event ($k = 0$). The initial single-mode squeezed vacuum state may be expressed in the Fock state basis as [14]

$$|\text{sq}\rangle = (1 - r^2)^{1/4} \sum_{n=0}^{\infty} \sqrt{\frac{(2n)!}{2^{2n}n!^2}} r^n |2n\rangle, \qquad (2)$$

where the squeezing parameter r, without loss of generality, is assumed to be real and nonnegative. The photon subtracted state is proportional to $\hat{a}|\text{sq}\rangle$, and its overlap with an odd cat state is

$$F_1 = \frac{|\langle\psi_{\text{cat}}^-|\hat{a}|\text{sq}\rangle|^2}{\langle\text{sq}|\hat{a}^\dagger\hat{a}|\text{sq}\rangle} = \frac{(1 - r^2)^{3/2}|\alpha|^2 e^{r\text{Re}(\alpha^2)}}{\sinh(|\alpha|^2)}. \qquad (3)$$

For a given desired cat size $|\alpha|$, the largest fidelity is obtained for real α and

$$r = r_1 \equiv \frac{\sqrt{9 + 4\alpha^4} - 3}{2\alpha^2}. \qquad (4)$$

r_1 and the corresponding maximized fidelity are plotted as functions of α in Fig. 1. When the cat size increases, the fidelity decreases and more squeezing is required. For cat states with $|\alpha| > 1.9$ the fidelity is below 0.9, and annihilation of a single photon from a squeezed vacuum state is thus not a suitable method to generate large cats as stated in the Introduction.

Turning now to the case of annihilation of three photons ($k = 1$), we find the odd cat state fidelity

$$
\begin{aligned}
F_3 &= \frac{|\langle\psi_{\text{cat}}^-|(\hat{a}^2 - \beta^2)\hat{a}|\text{sq}\rangle|^2}{\langle\text{sq}|\hat{a}^\dagger((\hat{a}^\dagger)^2 - (\beta^*)^2)(\hat{a}^2 - \beta^2)\hat{a}|\text{sq}\rangle} \\
&= \frac{(3r + r^2(\alpha^*)^2 - \beta^2)(3r + r^2\alpha^2 - (\beta^*)^2)}{|\beta|^4 - 3(\beta^2 + (\beta^*)^2)\frac{r}{1-r^2} + \frac{9r^2}{1-r^2} + \frac{15r^4}{(1-r^2)^2}} \\
&\quad \frac{(1 - r^2)^{3/2}|\alpha|^2 e^{r\text{Re}(\alpha^2)}}{\sinh(|\alpha|^2)}.
\end{aligned}
\qquad (5)
$$

Since we have assumed that r is real, it is optimal to choose α real, and in this case the fidelity is maximized for

$$r = r_3 \equiv \frac{\sqrt{\left(5 + \sqrt{10}\right)^2 + 4\alpha^4} - \left(5 + \sqrt{10}\right)}{2\alpha^2} \qquad (6)$$

and

$$\beta^2 = \beta_{\text{opt}}^2 \equiv \frac{3}{7 + 2\sqrt{10}}\alpha^2. \qquad (7)$$

A second local maximum exists for r and β^2 given by Eqs. (6) and (7) with $\sqrt{10}$ replaced by $-\sqrt{10}$. The optimized fidelity and r_3 are plotted in Fig. 1. The cat size, at which the fidelity drops below 0.90, is increased to $\alpha = 3.3$, and for $\alpha = \sqrt{6}$ the fidelity is 0.976. Note also that the
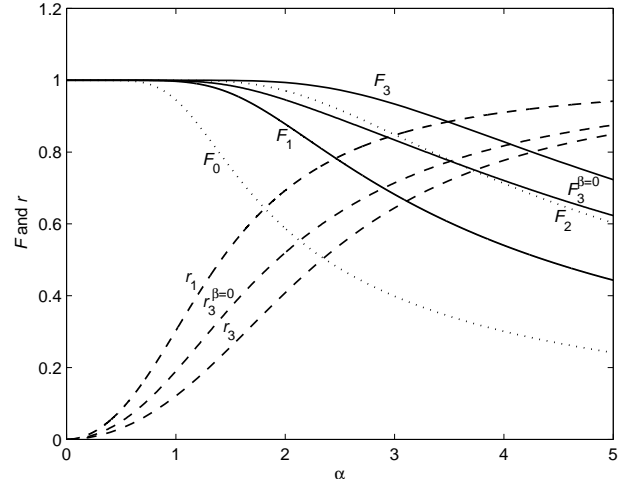

FIG. 1: The solid lines show the maximal odd cat state fidelity as a function of α for annihilation of a single photon (F_1, lower solid curve), annihilation of three photons (F_3, upper solid curve), and annihilation of three photons when $\beta = 0$ is assumed ($F_3^{\beta=0}$, middle solid curve). The dashed lines are the corresponding optimal values of r. The dotted lines give the maximal even cat state fidelity for the squeezed vacuum state (F_0, lower dotted line) and for annihilation of two photons (F_2, upper dotted line)
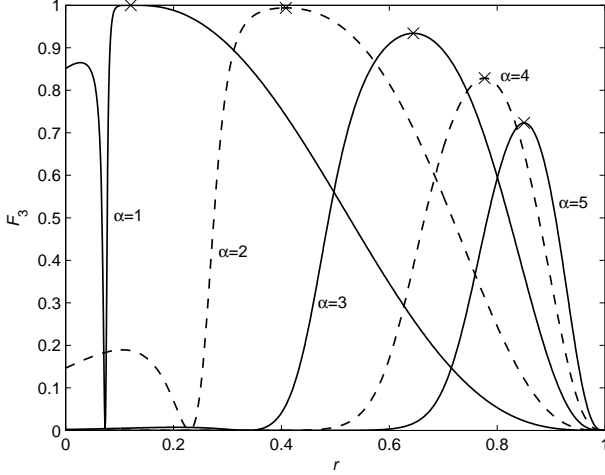
requested value of the degree of squeezing is decreased significantly compared to the case of a single count event.

Figure 2 shows the fidelity as a function of r for $\beta = \beta_{\text{opt}}$ and as a function of β for $r = r_3$ for a few values of α. This illustrates the consequences of small deviations from the optimal values of the parameters. It is, for instance, apparent that the parameters leading to $3r + r^2\alpha^2 - \beta^2 = 0$ and thus $F_3 = 0$ differ less from the optimal parameters for small values of α than for large values of α, and the obtained fidelity may thus be more sensitive to small deviations from the optimal parameters for small values of α, depending on the particular direction of the change. It is, however, much simpler to generate these high fidelity, small amplitude cats by subtracting only a single photon.

The experimental setup is less complicated if $\beta = 0$, and Fig. 1 shows that also in this case the maximal fidelity is increased compared to $k = 0$. For our reference cat size, $|\alpha| = \sqrt{6}$, the $\beta = 0$ fidelity yields $F_3^{\beta=0} = 0.90$ for the squeezing parameter $r_3^{\beta=0} = 0.62$.

Similar equations may be derived for even cat states, and the resulting maximal fidelities are also plotted in Fig. 1. It is apparent that the maximal odd cat fidelity following application of the operator $\hat{O}_3$, involving three annihilations, is larger than the maximal even cat state fidelity following the application of the operator $\hat{O}_2$, involving two annihilations, and we thus focus on odd cat state generation in the next section.
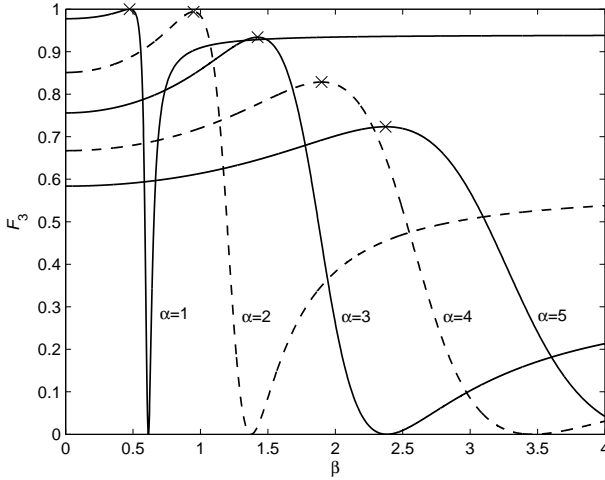
a)

b)

FIG. 2: Odd cat state fidelity for $2k + 1 = 3$ (a) as a function of r for $\beta = \beta_{\mathrm{opt}}$ and (b) as a function of β for $r = r_3$ for $\alpha = 1, 2, 3, 4, 5$. The maximal fidelity is marked with a cross on each line.

III. EXPERIMENTAL IMPLEMENTATION

The operator $\hat{O}_{2k+1}$ may ideally be implemented as shown for $2k + 1 = 3$ in Fig. 3. The first beam splitter sends a negligible small fraction of the initial squeezed vacuum state, generated by the OPO, onto a photo detector, and application of the first annihilation operator is obtained by conditioning on a photo detection event. The experiments for $2k + 1 = 1$ mentioned in the Introduction also use this method. The operator $\hat{a}^2 - \beta_i^2$ can be rewritten as $\hat{D}_{\hat{a}}^{-1}(-\beta_i)\hat{a}\hat{D}_{\hat{a}}(-\beta_i)\hat{D}_{\hat{a}}^{-1}(\beta_i)\hat{a}\hat{D}_{\hat{a}}(\beta_i)$, where $\hat{D}_{\hat{a}}(\beta_i) = e^{\beta_i \hat{a}^\dagger - \beta_i^* \hat{a}}$ is the field displacement operator [14], which may be implemented by mixing the state with a strong coherent state on a beam splitter with a
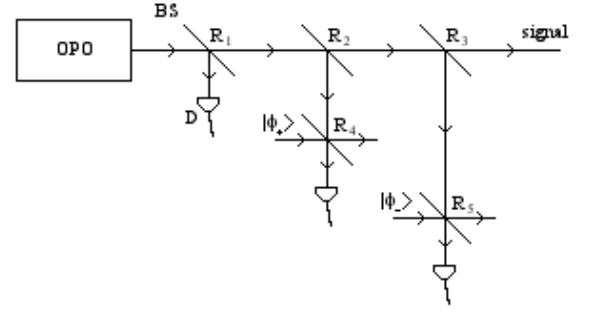

FIG. 3: Proposed experimental setup for generation of cat states conditioned on three photo detection events. OPO, pulsed degenerate optical parametric oscillator; BS, beam splitter; D, photo detector. $R_i \equiv 1 - T_i \ll 1$ is the reflectivity of beam splitter i, and $|\phi_\pm\rangle$ are input coherent states with amplitudes $\phi_+ = \sqrt{R_2/R_4}\beta$ and $\phi_- = -\sqrt{R_3/R_5}\beta$.

very small reflectivity [15]. To see this, we imagine a beam splitter with transmission τ and feed one input port with a coherent state $|\phi\rangle$ and the other with an arbitrary input state with density operator $\hat{\rho}_{\mathrm{in}}$. Let $\hat{a}$ denote the annihilation operator of the mode occupied by the input state and $\hat{b}$ the annihilation operator of the mode occupied by the coherent state. The action of the beam splitter is then represented by the unitary operator

$$\hat{U}_\tau = \exp\left(i \tan^{-1}\left(\sqrt{\frac{1-\tau}{\tau}}\right)\left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)\right), \quad (8)$$

and, after tracing out the $\hat{b}$ mode, we obtain the output state

$$
\begin{aligned}
\hat{\rho}_{\mathrm{out}} &= \frac{1}{\pi} \iint \langle\gamma|\hat{U}_\tau|\phi\rangle \hat{\rho}_{\mathrm{in}} \langle\phi|\hat{U}_\tau^\dagger|\gamma\rangle d\gamma_r d\gamma_i \\
&= \sqrt{\tau}^{\hat{a}^\dagger \hat{a}} e^{-\zeta^*\phi\hat{a}^\dagger + \tau\zeta\phi^*\hat{a}} \sum_{n=0}^{\infty} \frac{1}{n!}(1-\tau)^n \hat{a}^n \hat{\rho}_{\mathrm{in}}(\hat{a}^\dagger)^n \\
&\qquad e^{-\zeta\phi^*\hat{a} + \tau\zeta^*\phi\hat{a}^\dagger} \sqrt{\tau}^{\hat{a}^\dagger \hat{a}}, \quad (9)
\end{aligned}
$$

where $\gamma_r \equiv \mathrm{Re}(\gamma)$, $\gamma_i \equiv \mathrm{Im}(\gamma)$, $|\gamma\rangle$ is a coherent state, and $\zeta \equiv i\sqrt{(1-\tau)/\tau}$. In the limit $\sqrt{1-\tau} \to 0$, $\phi \to \infty$, and $\sqrt{1-\tau}\phi \to$ constant, (9) reduces to

$$\hat{\rho}_{\mathrm{out}} = \hat{D}_{\hat{a}}(i\sqrt{1-\tau}\phi)\hat{\rho}_{\mathrm{in}}\hat{D}_{\hat{a}}^\dagger(i\sqrt{1-\tau}\phi). \quad (10)$$

This analysis suggests that we separate a small fraction of the beam for the first single photon detection, we then displace the remaining field by the appropriate amplitude β and extract a fraction of that beam for photo detection, we subsequently displace the field towards the opposite amplitude $-\beta$ and extract again a small fraction for detection, and finally we apply a displacement to the remaining field. If all three photo detection events occur, the resulting state is given by the the desired $\hat{O}_3|\mathrm{sq}\rangle$. It is, however, not very efficient first to displace the state, then annihilate a photon, and finally displace it back again to

obtain $\hat{D}_{\hat{a}}^{-1}(\beta_i)\hat{a}\hat{D}_{\hat{a}}(\beta_i)$, and we suggest instead to displace only the small fractions of the field which are subject to photo detection. The inverse displacements are then not necessary. If we start with the state $\hat{\rho}_{\mathrm{in}}$, subtract a small fraction using a beam splitter with reflectivity $R = 1 - T$, displace this fraction by the amount $i\sqrt{R}\beta_i$ according to the above procedure, annihilate a photon in the displaced mode $\hat{b}$, and trace out the detected mode, we obtain the state

$$\hat{\rho}_{\mathrm{out}} \propto \sum_{n=0}^{\infty} \langle n|\hat{b}\hat{D}_{\hat{b}}(i\sqrt{R}\beta_i)\hat{U}_T|0\rangle\hat{\rho}_{\mathrm{in}}\langle 0|\hat{U}_T^\dagger\hat{D}_{\hat{b}}^\dagger(i\sqrt{R}\beta_i)\hat{b}^\dagger|n\rangle$$
(11)

for the transmitted field. For $R \ll 1$, we may expand (11) in orders of R, and to lowest order we find

$$\hat{\rho}_{\mathrm{out}} \propto R(\hat{a} + \beta_i)\rho_{\mathrm{in}}(\hat{a}^\dagger + \beta_i^*), \qquad (12)$$

which is precisely the desired result.

A drawback of this experimental implementation is that, in the ideal limit of zero beam splitter reflectivities, the success probability, i.e., the probability to obtain the trigger detection events in a given pulse of the setup, vanishes. It is thus necessary to allow nonzero reflectivities, but this compromises the desired output (12), as it effectively induces a loss from the output channel into the trigger channel. Large cat states are very sensitive to losses, and, in general, it is necessary to keep losses at an absolute minimum in order to obtain high fidelity, large amplitude cat states. To illustrate this point, we imagine the effect of sending a unit fidelity odd cat state of size $|\alpha|$ through a beam splitter with reflectivity $R = 1 - T$. After the beam splitter the overlap with a cat of size $|\alpha|$ is decreased to

$$\sum_{m=0}^{\infty} \langle m|\langle\psi_{\mathrm{cat}}^-|\hat{U}_T|\psi_{\mathrm{cat}}^-\rangle|0\rangle\langle 0|\langle\psi_{\mathrm{cat}}^-|\hat{U}_T^\dagger|\psi_{\mathrm{cat}}^-\rangle|m\rangle =$$
$$\frac{\cosh(R|\alpha|^2)\sinh^2(\sqrt{T}|\alpha|^2)}{\sinh^2(|\alpha|^2)}, \quad (13)$$

and for $|\alpha|^2 = 6$ and $R = 0.01$ this expression evaluates to 0.94. In the following we show that it is possible to obtain fidelities that are essentially equal to those given in Fig. 1 and simultaneously obtain acceptable success probabilities if the standard photo detectors are replaced by single photon number resolving detectors. The qualitative explanation for this for $\beta = 0$ is that larger beam splitter reflectivities are allowable if the detectors are able to weed out the instances where more than a single photon are reflected at one of the beam splitters. We note that, if losses are negligible, a single photon detector may be built from a large number of unit efficiency photo detectors as explained in [16].

If the photo detectors in Fig. 3 are single photon number resolving detectors, (11) is replaced by $|\psi_{\mathrm{out}}\rangle \propto \langle 1|\hat{D}_{\hat{b}}(i\sqrt{R}\beta_i)\hat{U}_T|0\rangle|\psi_{\mathrm{in}}\rangle$, and the conditional
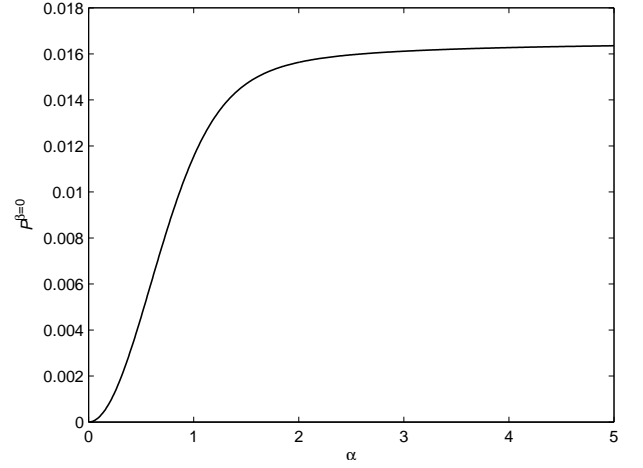

FIG. 4: Success probability as a function of α for the setup with three single photon number resolving trigger detectors and $\beta = 0$. The success probability is maximized under the constraint $x = r_3^{\beta=0}$ (see Fig. 1).

output state following three single photon detections is

$$|\psi_{\mathrm{out}}\rangle = \frac{1}{\sqrt{P}}\langle 1|\langle 1|\langle 1|\hat{D}_{\hat{b}_3}(-ir_3\beta)\hat{U}_{T_3}$$
$$\hat{D}_{\hat{b}_2}(ir_2\beta)\hat{U}_{T_2}\hat{U}_{T_1}|0\rangle|0\rangle|0\rangle|\psi_{\mathrm{in}}\rangle$$
$$= \frac{-i}{\sqrt{P}}\frac{r_1 r_2 r_3}{t_1 t_2^2 t_3^3}e^{\frac{1}{2}(r_2^2+r_3^2)|\beta|^2}\exp\left(\left(\frac{r_2^2}{t_2} - r_3^2\right)\frac{\beta^*}{t_3}\hat{a}\right)$$
$$(\hat{a}^2 - t_2 t_3^2 \beta^2 + (t_2 - 1)t_3\beta\hat{a})\hat{a}(t_3 t_2 t_1)^{\hat{a}^\dagger\hat{a}}|\psi_{\mathrm{in}}\rangle, \quad (14)$$

where P is the success probability, $r_i = \sqrt{R_i}$, $t_i = \sqrt{T_i}$, and $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ are annihilation operators of the three detected modes. Due to the exponential factor in $\hat{a}$ and the term $(t_2 - 1)t_3\beta\hat{a}$, the operator acting on the input state $|\psi_{\mathrm{in}}\rangle$ now also includes terms that annihilate an even number of photons if $\beta \neq 0$. For $\beta = 0$ and $|\psi_{\mathrm{in}}\rangle = |\mathrm{sq}\rangle$, on the other hand, the fidelity is again given by Eq. (5), except that r is replaced by $x \equiv rT_1T_2T_3$. Choosing $x = r_3^{\beta=0}$ we thus obtain the fidelities given by the middle solid curve in Fig. 1, and the beam splitter reflectivities may be chosen in order to maximize the success probability (see Eq. (16) below). The results of such an optimization are shown in Fig. 4, and $P^{\beta=0}$ is seen to be approximately $1.6 \cdot 10^{-2}$ for values of α around $\sqrt{6}$.

Despite the terms annihilating an even number of photons, the fidelity may be increased by choosing an optimized nonzero value of β^2. For $\beta \neq 0$ we can get rid of the exponential factor in (14), if we choose $r_3^2 = r_2^2/t_2$, and if R_2 is not too large, the term $(t_2 - 1)t_3\beta\hat{a}$ has little effect. For $r_3^2 = r_2^2/t_2$ and $|\psi_{\mathrm{in}}\rangle = |\mathrm{sq}\rangle$ the expression for

the odd cat state fidelity

$$F_3 = \frac{(1-x^2)^{3/2}|\alpha|^2 e^{x\mathrm{Re}(\alpha^2)}}{\sinh(|\alpha|^2)}(3x + x^2(\alpha^*)^2 - t_2 t_3^2 \beta^2)$$

$$(3x+x^2\alpha^2-t_2 t_3^2(\beta^*)^2)\Big/\Big(t_2^2 t_3^4|\beta|^4 - t_2 t_3^2(\beta^2+(\beta^*)^2)\frac{3x}{1-x^2}$$

$$+ (t_2-1)^2 t_3^2|\beta|^2\frac{1+2x^2}{1-x^2} + \frac{9x^2}{1-x^2} + \frac{15x^4}{(1-x^2)^2}\Big). \quad (15)$$

is almost identical to Eq. (5), but r is replaced by x, β^2 is replaced by $t_2 t_3^2 \beta^2$, and an extra term has been added in the denominator. Since this term will not influence the fidelity significantly for t_2 close to unity, we expect that it is nearly optimal to choose x and $t_2 t_3^2 \beta^2$ in accordance with Eqs. (6) and (7), respectively. If we choose T_2 such that the extra term is a factor of 10^{-3} smaller than the sum of the rest of the terms in the denominator, the fidelity is only decreased by approximately 0.1% compared to the fidelities represented by the upper solid curve in Fig. 1. The last parameter T_1 may now be chosen in order to maximize the success probability

$$P = \langle\psi_{\mathrm{out}}|\psi_{\mathrm{out}}\rangle = \frac{r_1^2 r_2^2 r_3^2}{t_1^2 t_2^4 t_3^6}e^{(r_2^2+r_3^2)|\beta|^2}\sqrt{\frac{1-r^2}{1-x^2}}$$

$$\Big(t_2^2 t_3^4|\beta|^4\frac{x^2}{1-x^2} + (t_2-1)^2 t_3^2|\beta|^2\frac{x^2(1+2x^2)}{(1-x^2)^2}$$

$$- 2t_2 t_3^2\mathrm{Re}(\beta^2)\frac{3x^3}{(1-x^2)^2} + \frac{3x^4(3+2x^2)}{(1-x^2)^3}\Big). \quad (16)$$

The optimal choice for fixed x, T_2, and T_3 is

$$T_1 = \frac{\sqrt{x^4 + 8T_2^2 T_3^2 x^2} - x^2}{2T_2^2 T_3^2} \quad (17)$$

(provided (17) leads to valid values of T_1 and r), and for $\alpha = \sqrt{6}$ we find $P = 6 \cdot 10^{-4}$. The price to pay for the increase in fidelity is thus a more complicated setup and a decrease in the success probability, but if the repetition rate of the experiment is around 10^6 s^{-1} (see Ref. [13]), it is still possible to produce of order 10^3 cat states per second.

The protocol suggested in Ref. [10] combines two cat states of the same size $|\alpha|$ on a beam splitter to obtain a superposition of a cat state of size $\sqrt{2}|\alpha|$ at one of the output ports and a vacuum state at the other or opposite site. A conditional measurement performed on one of the output beams projects the other beam on the cat state. With a single amplification, cats of size $|\alpha| = \sqrt{6}$ can thus be generated from cats of size $|\alpha| = \sqrt{3} \approx 1.73$. Figure 1 reveals, however, that the maximal fidelity of the cat states with $|\alpha| = \sqrt{3}$, generated by subtracting a single photon from a squeezed vacuum state, is only 0.93, and to achieve a higher fidelity one may instead start from smaller cats with larger fidelity and then amplify the states multiple times. With a single photon number resolving detector, the success probability to generate a cat state of size $|\alpha| = \sqrt{3/2}$ by detecting a single photon is $P = 0.13$, and for twofold amplification the probability to generate the initial four cat states simultaneously is thus $3 \cdot 10^{-4}$. Since the amplification protocol is itself probabilistic, the total success probability is approximately one or two orders of magnitude smaller (see [10]).

IV. CONCLUSION

The use of linear optics combined with measurements for quantum computing was proposed in [17], where single photon states had to be provided as an online resource. As pointed out in [5], sufficiently strong coherent states have several advantages over single photon states, but for qubit implementation one must have a means to provide Schrödinger cat states of a large enough amplitude to ensure that the two components $|\alpha\rangle$ and $|-\alpha\rangle$ of the cat state wave function are almost orthogonal.

In this paper we have suggested a protocol that is suitable to generate such high fidelity cat states from the output of an OPO. The states are heralded by three joint photo detection events. Using normal APD photon counters, high fidelity states may only be obtained with very small success probability, but with detectors that can discriminate a single photon from higher photon numbers, we predict quite acceptable production yields of high fidelity states.

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